1e

The coefficient of ~~[illegible]~~ $\langle x | n \rangle = f_n(x)$ is a basis wavefunction of the harmonic oscillator

quantum number

thing projected on

The expansion of a wavefunction $\psi(x) = \langle x | \psi \rangle = \sum_n \langle x | n \rangle \langle n | \psi \rangle = \sum_n c_n f_n(x)$

1c

$$\hat{a}_+ + \hat{a}_- = \frac{1}{\sqrt{2\hbar m\omega}}\left(2m\omega\hat{x}\right) = \sqrt{\frac{2m\omega}{\hbar}}\,\hat{x}$$

$$\text{or } \hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}_+ + \hat{a}_-\right)$$

$$\hat{a}_+ - \hat{a}_- = \frac{1}{\sqrt{2\hbar m\omega}}\left(-i\hat{p} - i\hat{p}\right)$$

$$= \frac{-2i}{\sqrt{2\hbar m\omega}}\,\hat{p} \Rightarrow \hat{p} = -i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a}_+ - \hat{a}_-\right)$$

$$= i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a}_+ - \hat{a}_-\right)$$

2a

$$H = -\gamma(B \cdot S) = -\gamma\left(B_0 \cos\omega t\, \hat{k} \cdot \frac{\hbar}{2}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{pmatrix}\right) = -\gamma B_0 \cos(\omega t)\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

2b

time-dependent Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi$$

and $E\Psi = H\Psi$

but also

$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$; substituting H and replacing Ψ by χ, we get

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} = -\frac{\gamma B_0 \hbar}{2}\cos(\omega t)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix}$$

Simplifying then gives

$$\frac{\partial}{\partial t}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} = -\frac{\gamma B_0}{2i}\cos(\omega t)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} \quad \square$$

2c

We get 2 equations:

$$\frac{\partial}{\partial t}\alpha(t) = -\frac{\gamma B_0}{2i}\cos(\omega t)\,\alpha(t)$$

$$\frac{\partial}{\partial t}\beta(t) = \frac{\gamma B_0}{2i}\cos(\omega t)\,\beta(t)$$

separation of variables gives

$$\frac{1}{\alpha(t)}d\alpha(t) = -\frac{\gamma B_0}{2i}\cos(\omega t)\,dt \rightarrow \ln(\alpha(t)) = -\frac{\gamma B_0}{2\omega i}\sin(\omega t) + C_1$$

$$\frac{1}{\beta(t)}d\beta(t) = \frac{\gamma B_0}{2i}\cos(\omega t)\,dt \rightarrow \ln(\beta(t)) = \frac{\gamma B_0}{2\omega i}\sin(\omega t) + C_2$$

$$\alpha(t) = e^{\frac{\gamma B_0 i}{2\omega}\sin(\omega t)}e^{C_1}$$

$$\beta(t) = e^{-\frac{i\gamma B_0}{2\omega}\sin(\omega t)}e^{C_2}$$

for $t=0$, the sines are 0; we get $e^{C_1} = e^{C_2} = \frac{1}{\sqrt{2}}$ using the boundary conditions

this is the desired result given

2d

$$\sigma_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \lambda^2 - \left(\frac{\hbar}{2}\right)^2 = 0$$

$$\lambda^2 = \frac{\hbar^2}{2^2} \Rightarrow \lambda = \pm \frac{\hbar}{2}$$

$$\begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \pm\frac{\hbar}{2}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\frac{\hbar}{2}\beta = \pm\frac{\hbar}{2}\alpha \Rightarrow \beta = \pm\alpha \Rightarrow \text{eigenvectors } \begin{pmatrix} \alpha \\ \alpha \end{pmatrix} \text{ and } \begin{pmatrix} \alpha \\ -\alpha \end{pmatrix}$$

$$\frac{\hbar}{2}\alpha = \pm\frac{\hbar}{2}\beta$$

normalizing gives

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \text{ and } \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = ?_1\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + ?_2\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$a = \frac{?_1 + ?_2}{\sqrt{2}} \Rightarrow ?_1 + ?_2 = \sqrt{2}\,a$$

$$b = \frac{?_1 - ?_2}{\sqrt{2}} \Rightarrow ?_1 - ?_2 = \sqrt{2}\,b$$

$$2?_1 = (a+b)\sqrt{2}$$

$$?_1 = \frac{a+b}{\sqrt{2}}$$

$$?_2 = \sqrt{2}a - ?_1 = \frac{2a - a - b}{\sqrt{2}} = \frac{a-b}{\sqrt{2}}$$

We thus want to know $\left|\left(\frac{a-b}{\sqrt{2}}\right)\right|^2 = \left|\left(\frac{e^{i\overbrace{(\gamma B_0/2\omega)\sin(\omega t)}^{c}} - e^{-i(\gamma B_0/2\omega)\sin(\omega t)}}{2}\right)\right|^2$

$$= \left|\left(\frac{\cos(c) + i\sin(c) - \cos(c) - -i\sin(c)}{2}\right)\right|^2$$

$$= \left|\left(\frac{2i\sin c}{2}\right)\right|^2 = |i^2|\sin^2 c = |-1|\sin^2 c$$

$$= \sin^2 c = \sin^2\left(\frac{\gamma B_0}{2\omega}\sin(\omega t)\right)$$

$\square$

3a

strong - field Zeeman effect

$$H = H_{Bohr} + H_Z'$$

$$H_Z' = \frac{e}{2m} B_{ext} (\hat{L}_z + 2\hat{S}_z)$$, as Bext is only in the z- direction

In the strong field Zeeman effect: The good quantum numbers are principal quantum number n, l, m_s and m_l

$$E_{n l m_s m_l} = \langle H_{Bohr} + H' \rangle = \langle n l m_l m_s | H_{Bohr} | n l m_l m_s \rangle \rightarrow -\frac{13.6\,eV}{n^2}$$

$$+ \langle n l m_l m_s | H_z' | n l m_l m_s \rangle$$

$$\rightarrow \frac{e}{2m} B_{ext} (\underset{\text{ev of } L_z}{m_l} + 2 \underset{\text{ev of } S_z}{m_s})$$

3b

$$E_{so}' = \langle n l m_l m_s | H_{so}' | n l m_l m_s \rangle$$

commutes with $\hat{S}\cdot\hat{L}$

$$= \frac{e^2}{8\pi\varepsilon_0} \frac{1}{m^2 c^2} \langle n l m_l m_s | \frac{1}{r^3} \hat{S}\cdot\hat{L} | n l m_l m_s \rangle$$

$$\Rightarrow E_{so}' = \frac{e^2}{8\pi\varepsilon_0 m^2 c^2 \, l(l+\frac{1}{2})(l+1) n^3 a^3} \langle n l m_l m_s | \hat{S}\cdot\hat{L} | n l m_l m_s \rangle$$

note that $|n l m_l m_s\rangle$ is an eigenstate of both the L_z and S_z operators;

$$\langle S_x \rangle = \langle S_y \rangle = \langle L_x \rangle = \langle L_y \rangle = 0$$

$$\langle n l m_l m_s | S_z L_z | n l m_l m_s \rangle = m_l m_s$$

$$E_{so}' = \frac{e^2}{8\pi\varepsilon_0 m^2 c^2} \frac{m_l m_s}{l(l+\frac{1}{2})(l+1) n^3 a^3}$$

$\boxed{3C}$ $n=2,\ l \in \{0, 1\},\ s=\frac{1}{2}$

$j =$ "everything in integer steps between $s+l$ and $|l-s|$"

$$\left\{\begin{array}{l} j \in \{s+l,\ s+l-1,\ \dots,\ |l-s|\} \\ m_l = -l, \dots, +l \\ m_s = -s, \dots, +s \\ m_j = -j, \dots, +j \end{array}\right.$$

$l = 0, 1 \rightarrow m_l = 1, 0, -1$

$s = \frac{1}{2} \rightarrow m_s = -\frac{1}{2}, \frac{1}{2}$

$j = \frac{3}{2}, \frac{1}{2} \rightarrow m_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$

case A: $l = 1,\ s = \frac{1}{2}$

also, we need $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

$|j\ m_j\rangle \qquad |l\ m_l\rangle\,|s\ m_s\rangle$

$\left|\frac{3}{2}\ \frac{3}{2}\right\rangle = |1\ 1\rangle \left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\,|1\ 1\rangle \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\,|1\ 0\rangle \left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

$\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle =$

$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle = |1\ -1\rangle \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$

$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\,|1\ 0\rangle \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\,|1\ -1\rangle \left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

$\boxed{3c \mid part\ 2}$ $l=0$ $s=\frac{1}{2}$

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle = |0\ 0\rangle \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle = |0\ 0\rangle \left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle$$

3d

given: $E_{fs}^1 = \frac{(E_n)^2}{2mc^2}\left(3 - \frac{4n}{j+\frac{1}{2}}\right) = \frac{(E_n)^2}{2mc^2}\left(3 - \frac{4n}{3}\right)$

$j = \frac{5}{2}$

$\cancel{n=2}$

$= \frac{(E_n)^2}{2mc^2}\left(3 - \frac{4n}{3}\right)$

$E_z^1 = \langle n l j m_j | H_z^1 | n l j m_j \rangle$

with $H_z^1 = \frac{e}{2m}(L_z + 2S_z)B_{ext}$

We have the state $|\frac{5}{2}\ \frac{1}{2}\rangle$

insert and compute eigenvalues

$|2\ 1\rangle |\frac{1}{2}\ -\frac{1}{2}\rangle$

$E_z^1 = \frac{e}{2m} B_{ext}\Big(\frac{2}{5} \langle 21| \langle \frac{1}{2}\ -\frac{1}{2}| L_z + 2S_z$

$+ \frac{3}{5} \langle 20| \langle \frac{1}{2}\ \frac{1}{2}| L_z + 2S_z |20\rangle |\frac{1}{2}\ \frac{1}{2}\rangle \Big)$

$= \frac{eB_{ext}}{2m}\left(\frac{2}{5}(1-1) + \frac{3}{5}(0+1)\right) = \frac{3\ eB_{ext}}{10\ m}$

total first order - correction: $E^1 = E_{fs}^1 + E_z^1 = \cancel{\frac{(E_2)^2}{6mc^2}} + \frac{3e}{10m}B_{ext}$

$\frac{(E_n)^2}{2mc^2}\left(3 - \frac{4n}{3}\right)$

$\frac{1}{2} \times \frac{1}{2}$

l

$m_s = s_1 + s_2$

		1
		+1
$+\frac{1}{2}$	$+\frac{1}{2}$	1

projection spin 1 $= m_1$

projection spin 2 $= m_2$

total ang. mom. l

total projection $= 0$

		1	0
		0	0
$+\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$-\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

ang. mom. spin

$1 \times \frac{1}{2}$

3e

$H_s' = eE_{ext} z = eE_{ext} r\cos\theta$

spherical coordinates

$\psi_0(r) = \frac{1}{\sqrt{\pi a^3}} e^{-\frac{r}{a}} \Rightarrow E_s' = \langle \psi_0 | H_s' | \psi_0 \rangle$

$= \int d^3r \, \psi_0 H_s' \psi_0 =$

$$\underbrace{2\pi}_{\int_0^{2\pi} d\phi} \underbrace{\int_0^\pi d\theta \cos\theta \sin\theta}_{=0} \int_0^\infty dr \, r^3 \frac{1}{\pi a^3} e^{-2r/a} = 0$$

3f

$$E_n^2 = \sum_{m \neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}$$

$\langle nlm | z | 100 \rangle$

$\langle 200 | z | 100 \rangle = 0$

$\langle 21{-1} | z | 100 \rangle = 0$

$\langle 210 | z | 100 \rangle = \langle \frac{1}{2\sqrt{6}} a^{-\frac{3}{2}} \frac{r}{a} e^{-\frac{r}{2a}} \left(\frac{3}{2\pi}\right)^{\frac{1}{2}} \cos\theta | \underset{r\cos\theta}{z} | \frac{1}{\sqrt{\pi a^3}} e^{-\frac{r}{a}} \rangle$

$\langle 211 | z | 100 \rangle = 0$

selection rules

$\Delta l = \pm 1$

$\Delta m \in \{0, 1, -1\}$

$\langle 210 | z | 100 \rangle = eE_{ext} \cdot \ldots$

We also need to calculate $E_1^0 - E_2^0 = E_1 \left(1 - \frac{1}{4}\right) = \frac{3}{4} E_1$

$$\Rightarrow E_1^2 = \frac{e^2 E_{ext}^2 a^2 \left(\frac{2}{3}\right)^{10} \cdot 32 \cdot}{\frac{3}{4} E_1}$$

4a

$$E_n = \langle \psi_n | H | \psi_n \rangle$$

$$\frac{\partial E_n}{\partial \lambda} = \left\langle \frac{\partial \psi_n}{\partial \lambda} \right| H | \psi_n \rangle + \left\langle \psi_n \right| \frac{\partial H}{\partial \lambda} \left| \psi_n \right\rangle + \left\langle \psi_n \right| H \left| \frac{\partial \psi_n}{\partial \lambda} \right\rangle$$

$$= E_n \left\langle \frac{\partial \psi_n}{\partial \lambda} \right| \psi_n \rangle + \left\langle \psi_n \right| \frac{\partial H}{\partial \lambda} \left| \psi_n \right\rangle + E_n \left\langle \psi_n \right| \left. \frac{\partial \psi_n}{\partial \lambda} \right\rangle$$

$$= \underbrace{2 E_n \frac{\partial}{\partial \lambda} \underbrace{\langle \psi_n | \psi_n \rangle}_{1}}_{0} + \left\langle \psi_n \right| \frac{\partial H}{\partial \lambda} \left| \psi_n \right\rangle$$

$$= \qquad + \left\langle \psi_n \right| \frac{\partial H}{\partial \lambda} \left| \psi_n \right\rangle \quad \square$$

4b

$$\lambda = \hbar$$

$$\frac{\partial H}{\partial \lambda} = \frac{2\hbar}{2m} \frac{d^2}{dx^2}$$

$$\frac{\partial E_n}{\partial \hbar} = \left\langle \psi_n \right| \frac{\partial H}{\partial \hbar} \left| \psi_n \right\rangle = \left\langle \psi_n \right| \frac{\hbar}{m} \frac{d^2}{dx^2} \left| \psi_n \right\rangle$$

$$\frac{\hbar}{2} \frac{\partial E_n}{\partial \hbar} = \left\langle \psi_n \right| \frac{\hbar^2}{2m} \frac{d^2}{dx^2} \left| \psi_n \right\rangle$$

$$\frac{\hbar}{2} \frac{\partial E_n}{\partial \hbar} = \langle T \rangle$$

$$\langle T \rangle = \frac{\hbar \omega}{2} \left(n + \frac{1}{2} \right)$$

4b | part2

$\lambda = \omega$

$$\frac{\partial H}{\partial \lambda} = \frac{\partial H}{\partial \omega} = m\omega x^2$$

$$\frac{\partial E_n}{\partial \lambda} = \left\langle \psi_n \right| \frac{\partial H}{\partial \omega} \left| \psi_n \right\rangle = \left\langle \psi_n \right| m\omega x^2 \left| \psi_n \right\rangle$$

$$\frac{1}{2}\omega \frac{\partial E_n}{\partial \omega} = \left\langle \psi_n \right| \tfrac{1}{2} m\omega^2 x^2 \left| \psi_n \right\rangle = \langle V \rangle$$

using $E_n = (n+\frac{1}{2})\hbar\omega \rightarrow \langle V \rangle = \frac{1}{2}\omega\,(n+\frac{1}{2})\hbar = \frac{1}{2}\hbar\omega\,(n+\frac{1}{2})$, thus, we seem to have $\langle T \rangle = \langle V \rangle = \frac{1}{2}E_n$

4c

$$\lambda = m \rightarrow \frac{\partial H}{\partial \lambda} = \frac{\partial H}{\partial m} = +\frac{\hbar^2}{m^2} \frac{d^2}{dx^2} + \frac{1}{2}\omega^2 x^2$$

$$= \frac{1}{m}\langle V - T \rangle$$

$$\frac{\partial E_n}{\partial m} = 0$$

$$\downarrow$$

$$\langle V - T \rangle = 0$$

$$\downarrow$$

~~virial~~ virial theorem for harmonic oscillator: $\langle V \rangle = \langle T \rangle$

4d

Choose $\lambda = \frac{e^2}{4\pi\varepsilon_0}$

Then $\frac{\partial H}{\partial \lambda} = -\frac{1}{r}$, or

$$\frac{\partial E_n}{\partial \lambda} = \left\langle \psi_n \left| \frac{\partial H}{\partial \lambda} \right| \psi_n \right\rangle = \left\langle \psi_n \left| -\frac{1}{r} \right| \psi_n \right\rangle = \left\langle -\frac{1}{r} \right\rangle$$

$$= -\left\langle \frac{1}{r} \right\rangle$$

$$\left\langle \frac{1}{r} \right\rangle = -\frac{\partial E_n}{\partial \lambda} = -\frac{\partial}{\partial \frac{e^2}{4\pi\varepsilon_0}} \left(-\frac{me^4}{32\pi^2\varepsilon_0^2 \hbar^2 (j_{max}+l+1)^2} \right)$$

$$= \frac{\partial}{\partial \frac{e^2}{4\pi\varepsilon_0}} \left(\frac{m}{2\hbar^2 (j_{max}+l+1)^2} \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 \right)$$

$$= \frac{2\, me^2}{8\pi\varepsilon_0 \hbar^2 (j_{max}+l+1)^2} = \frac{2me^2}{8\pi\varepsilon_0 \hbar^2 n^2}$$

4e $(j_{max}+l+1)^2 = j_{max}^2 + l^2 + 1 + 2j_{max}l + 2j_{max} + 2l$

choose $\lambda = l$

Then $\frac{\partial H}{\partial \lambda} = \frac{\hbar^2}{2mr^2}(2l+1)$

$$\frac{\partial E_n}{\partial \lambda} = \frac{\partial E_n}{\partial l} = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{-(2l+2j_{max}+2)}{(j_{max}+l+1)^4}$$

$$= \frac{me^4}{16\pi^2\varepsilon_0^2\hbar^2(j_{max}+l+1)^3}$$

So, $\frac{me^4}{16\pi^2\varepsilon_0^2\hbar^2(j_{max}+l+1)^3} = \langle\psi_n|\frac{\hbar^2}{2mr^2}(2l+1)|\psi_n\rangle$

$$= \langle\psi_n|\frac{1}{r^2}|\psi_n\rangle\frac{\hbar^2(2l+1)}{2m}$$

or $\langle\frac{1}{r^2}\rangle = \frac{m^2e^4\cancel{(2l+1)}}{8\pi^2\hbar^4\varepsilon_0^2(j_{max}+l+1)^3(2l+1)}$ $-\frac{1}{2}$

? can we do this?!

4a

$H|\psi_n\rangle = E_n|\psi_n\rangle$ (1 point)

expectation value of $\langle\psi_n|H|\psi_n\rangle = E_n$

$$\frac{\partial E_n}{\partial\lambda} = \left\langle\frac{\partial\psi_n}{\partial\lambda}\right| H |\psi_n\rangle + \langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_n\rangle + \langle\psi_n|H\left|\frac{\partial\psi_n}{\partial\lambda}\right\rangle$$

$$\cancel{\left\langle\frac{\partial\psi_n}{\partial\lambda}\right| E |\psi_n\rangle + \langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_n\rangle}$$

$$= E_n\frac{d}{d\lambda}\underbrace{\langle\psi_n|\psi_n\rangle}_{1} + \langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_n\rangle + E_n\langle\psi_n|\left|\frac{\partial\psi_n}{\partial\lambda}\right\rangle$$

$$= \langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_n\rangle$$

4b $\lambda = \omega$

$$\left.\begin{aligned}\frac{\partial E_n}{\partial\omega} &= \hbar\left(n+\tfrac{1}{2}\right)\\ \frac{\partial H}{\partial\omega} &= m\omega x^2\end{aligned}\right\} \longrightarrow \langle V\rangle = \frac{1}{2}\omega\left\langle\frac{\partial H}{\partial\omega}\right\rangle = \frac{1}{2}\hbar\omega\left(n+\tfrac{1}{2}\right) = \frac{1}{2}E_n$$

$$\boxed{E_n = \hbar\omega\left(n+\tfrac{1}{2}\right) \quad (1\text{ point})}$$

$\lambda = \hbar$

$$\frac{\partial E_n}{\partial\hbar} = \omega\left(n+\tfrac{1}{2}\right)$$

$$\frac{\partial H}{\partial\hbar} = -\frac{\hbar}{m}\frac{d^2}{dx^2} \Longrightarrow \langle T\rangle = \frac{1}{2}\hbar\left\langle\frac{\partial H}{\partial\hbar}\right\rangle = \frac{1}{2}E_n$$

3e

$$E_1^1 = \langle \psi_1^0 | e E_{ext} z | \psi_1^0 \rangle \qquad z =$$

$$= \int \sqrt{\frac{1}{4\pi}}\, 2a^{-\frac{3}{2}} e^{-\frac{r}{a}}\, e E_{ext} z \sqrt{\frac{1}{4\pi}}\, 2a^{-\frac{3}{2}} e^{-\frac{r}{a}}\, dr\, d\theta\, d\phi$$

$$= \frac{1}{4\pi} 2^2 a^{-\frac{6}{2}}\, e_0 E_{ext} \int e^{-\frac{2r}{a}}$$