11.10 The transition rate for (thermally) stimulated emission is given by equation 11.54:

$$R_{b\to a} = \frac{\pi}{3\epsilon_0 \hbar^2} |\wp|^2 \rho(\omega_0)$$

The spontaneous emission rate is $A = \frac{\omega_0^3 |\wp|^2}{3\pi \epsilon_0 \hbar c^3}$

The ratio of these is

$$\frac{A}{R} = \frac{\omega_0^3 |\wp|^2}{3\pi\epsilon_0 \hbar c^3} \frac{3\epsilon_0 \hbar^2}{\pi |\wp|^2 \rho(\omega_0)} = \frac{\omega_0^3}{\pi^2} \frac{\hbar}{c^3 \rho(\omega_0)}$$

Substituting equation 11.59 gives

$$\frac{A}{R} = \frac{\omega_0^3 \hbar}{\pi^2 c^3} \frac{\pi^2 c^3}{\hbar \omega_0^3} \left(e^{\hbar\omega/k_B T} - 1\right) = e^{\hbar\omega/k_B T} - 1$$

this ratio is greater than 1 when $e^{\hbar\omega/k_B T} > 2$;

that is, when $\frac{\hbar\omega}{k_B T} > \ln(2)$

$$\omega > \frac{k_B T}{\hbar} \ln(2), \text{ or}$$

$\nu = \frac{\omega}{2\pi} = \frac{k_B T}{\hbar} \ln 2$; for $T = 300$ K, this results in

$$\nu = \frac{1.38 \times 10^{-23} \cdot 300}{6.63 \times 10^{-34}} \ln 2 \simeq 4.35 \times 10^{12}\ Hz$$

For higher frequencies (including visible light, at 10^{14} Hz), spontaneous emission dominates.

For lower frequencies, stimulated emission dominates