9.3

$$T \sim e^{-2\gamma}$$

$$\gamma = \frac{1}{\hbar}\int_0^{2a} |p(x)|\,dx$$

$$= \frac{1}{\hbar}\int_0^{2a} \left|\sqrt{2m(E-V(x))}\right| dx$$

$$= \frac{1}{\hbar}\int_0^{2a} \left|\sqrt{2m(E-V_0)}\right| dx \qquad (V_0 > E)$$

$$= \frac{\sqrt{2m}}{\hbar}\int_0^{2a} \sqrt{V_0 - E}\,dx \qquad (\text{integrand constant})$$

$$= \frac{2a\sqrt{2m(V_0-E)}}{\hbar}$$

This gives $T \sim e^{-\frac{4a}{\hbar}\sqrt{2m(V_0-E)}}$

Earlier, the (exact) result was $T = \dfrac{1}{1 + \dfrac{V_0^2}{4E(V_0-E)}\sinh^2\gamma}$

In the WKB approximation, it is assumed that T is small ($T \ll 1$), or, put differently, γ is large.

Then, $\sinh\gamma = \frac{1}{2}(e^{\gamma} - e^{-\gamma}) \approx \frac{1}{2}e^{\gamma}$

and $\sinh^2\gamma \approx \frac{1}{4}e^{2\gamma}$; it follows that

$$T \approx \frac{1}{1 + \frac{V_0^2}{16E(V_0-E)}e^{2\gamma}} \underset{\text{as } \frac{V_0^2}{4E(V_0-E)}e^{2\gamma} \gg 1}{\approx} \frac{1}{\frac{V_0^2}{16E(V_0-E)}}e^{-2\gamma} = \underbrace{\frac{16E(V_0-E)}{V_0^2}}_{=O(1)\text{ so}} e^{-2\gamma}$$

$T \approx e^{-2\gamma}$ ∴ the WKB approximation holds