9.1
part 1

$$V(x) = \begin{cases} V_0 & (0 < x < \frac{a}{2}) \\ 0 & (\frac{a}{2} < x < a) \\ \infty & (\text{otherwise}) \end{cases}$$

use WKB approximation to find allowed energies

$$\psi(x) = \frac{C_{\{1,2\}}}{\sqrt{p(x)}} e^{\pm \frac{i}{\hbar} \int p(x) dx}$$

$$p(x) = \sqrt{2m(E - V(x))}$$

$$\int \sqrt{2m(E - V(x))}\, dx = \ldots ?$$

or, writing the complex exponentials as trigonometric functions:

$$\psi(x) = \frac{C_3}{\sqrt{p(x)}} \sin(\phi(x)) + \frac{C_4}{\sqrt{p(x)}} \cos(\phi(x))$$

at $x=0$ (for which $\phi(0)=0$) and at $x=a$, boundary conditions related to the infinite potential imply that $\psi(0) = \psi(a) = 0$

From $\psi(0)=0$, it follows that $\frac{C_4}{\sqrt{p(x)}} \cos(0) = 0$, or $C_4 = 0$

Then, from $\psi(a)=0$, we get

$$\psi(a) = 0 = \frac{C_3}{\sqrt{p(x)}} \sin(\phi(a)) \rightarrow \phi(a) = n\pi$$

9.1 part 2

now, as $\phi(x) = \frac{1}{\hbar}\int_0^x p(x')\,dx'$, we get

$$\frac{1}{\hbar}\int_0^a p(x')\,dx' = n\pi$$

$$\int_0^a p(x')\,dx' = n\pi\hbar$$

with positive natural number n

now, $\int_0^a p(x')\,dx = \int_0^{\frac{a}{2}} p(x')\,dx + \int_{\frac{a}{2}}^a p(x')\,dx$

$$= \int_0^{\frac{a}{2}} \sqrt{2mE}\,dx' + \int_{\frac{a}{2}}^a \sqrt{2m(E-V_0)}\,dx$$

$$= \sqrt{2mE}\,\frac{a}{2} + \sqrt{2m(E-V_0)}\,a - \sqrt{2m(E-V_0)}\,\frac{a}{2}$$

$$= \sqrt{2mE}\,\frac{a}{2} + \sqrt{2m(E-V_0)}\,\frac{a}{2}$$

$$= \sqrt{2m}\,\frac{a}{2}\left(\sqrt{E} + \sqrt{E-V_0}\right) = n\pi\hbar$$

$$\frac{n\pi\hbar\,2}{\sqrt{2m}\,a} = \sqrt{E} + \sqrt{E-V_0}$$

squaring both sides gives

$$\frac{4n^2\pi^2\hbar^2}{2ma^2} = E + E - V_0 + 2\sqrt{E(E-V_0)}$$

$$4E_n^0 = \frac{4(n\pi\hbar)^2}{2ma^2} = 2E - V_0 + 2\sqrt{E(E-V_0)}$$

$$4E_n^0 - 2E + V_0 = 2\sqrt{E(E-V_0)}$$

squaring both sides again gives

$$-4EV_0 + 16(E_n^0)^2 + 4E^2 + V_0^2 - 16EE_n^0 + 8E_n^0V_0 = 4E(E-V_0)$$

9.1
part 3

$$16(E_n^0)^2 + V_0^2 + 8E_n^0 V_0 = 4E^2 - 4EV_0 + 4EV_0 - 4E^2 + 16EE_n^0$$

$$16(E_n^0)^2 + V_0^2 + 8E_n^0 V_0 = 16EE_n^0$$

$$E = E_n^0 + \frac{1}{2}V_0 + \frac{V_0^2}{16E_n^0}$$

In perturbation theory, the result was

$E_n = E_n^0 + \frac{V_0}{2}$; the additional term $\left(\frac{V_0^2}{16E_n^0}\right)$ goes to zero for small V_0 or large n (as $E_n^0 \sim n^2$)