11.2

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}} e^{-\frac{r}{a}} \qquad \psi_{200} = \frac{1}{\sqrt{8\pi a^3}}\left(1-\frac{r}{2a}\right)e^{-\frac{r}{2a}}$$

$$\psi_{210} = \frac{1}{\sqrt{32\pi a^3}} \frac{r}{a} e^{-\frac{r}{2a}} \cos\theta \qquad \psi_{21\pm1} = \mp \frac{1}{\sqrt{64\pi a^3}} \frac{r}{a} e^{\frac{r}{2a}} \sin\theta\, e^{\pm i\phi}$$

here: $r\cos\theta = z$

and $r\sin\theta e^{\pm i\phi} = r\sin\theta(\cos\phi \pm i\sin\phi) = r\sin\theta\cos\phi \pm ir\sin\theta\sin\phi = x \pm iy$

Then, it follows that $|\psi|^2$ is an even function of z in all cases; thus, $\int z|\psi|^2 dx\,dy\,dz = 0$, so $H'_{ii}=0$

(as $H'_{ii} = \langle\psi_i|H'|\psi_i\rangle = \langle\psi_i|eEz|\psi_i\rangle$)

Also, ψ_{100} is even in z, ψ_{200} is even in z, ψ_{211} is even in z and ψ_{21-1} is even in z; so, only one matrix element H'_{ij} is unequal to zero (alternatively use the selection rules, $n' \neq n$, $l'-l = \pm1$, $m'-m \in \{0,1,-1\}$)

$$H'_{100,210} = -eE\frac{1}{\sqrt{32\pi^2 a^6}\,a}\int e^{-\frac{r}{a}}e^{-\frac{r}{2a}} r\cos\theta\, d^3\vec{r} = \frac{-eE}{4\sqrt{2}\pi a^4}\int e^{-\frac{3r}{2a}} r^2\cos^2\theta\, r^2\sin\theta\, dr\,d\theta\,d\phi$$

$$= -\frac{eE}{4\sqrt{2}\pi a^4}\underbrace{\int_0^\infty r^4 e^{-\frac{3r}{2a}}dr}_{\text{exp. integral: } n=4,\ \alpha=\frac{2a}{3}}\ \underbrace{\int_0^\pi \cos^2\theta\sin\theta\,d\theta}_{=\frac{2}{3}}\int_0^{2\pi} d\phi$$

$u=\cos(x)$, $du = \sin(x)dx$ $(x=\theta)$

$\left(=-\int_1^{-1} u^2 du = \int_{-1}^{1} u^2 du = \frac{1}{3}\left(1^3-(-1)^3\right) = \frac{2}{3}\right)$

$$= -\frac{eE}{4\sqrt{2}\pi a^4}\, 4!\left(\frac{2a}{3}\right)^5 \frac{2}{3}\, 2\pi = -\left(\frac{2^8}{3^5\sqrt{2}}\right)eEa$$

$$\simeq -0,7449\, eEa$$