Exercise 7.2

Tuesday, 29 September 2020 10:09

$V(x) = \frac{1}{2} k x^2$

a) $E_n = \left(n + \frac{1}{2}\right) \hbar \omega$

$\omega = \sqrt{\frac{k}{m}}$

We change $k \to (1+\varepsilon) k$

$f' = \frac{1}{2}(1+\varepsilon)^{-\frac{1}{2}}$ $\quad f'' = -\frac{1}{4}(1+\varepsilon)^{-\frac{3}{2}}$

series: $1 + \frac{1}{2} \varepsilon + \frac{1}{2} \cdot -\frac{1}{4} \varepsilon^2$

$$E_{n,new} = \left(n+\frac{1}{2}\right) \hbar \omega' = \left(n+\frac{1}{2}\right)\hbar \sqrt{\frac{k(1+\varepsilon)}{m}} = \left(n+\frac{1}{2}\right)\hbar \sqrt{\frac{k}{m}} \sqrt{1+\varepsilon} \approx \left(n+\frac{1}{2}\right)\hbar \sqrt{\frac{k}{m}} \left(1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \ldots\right)$$

---

b) $$H' = H - H_0 = \frac{1}{2} k' x^2 - \frac{1}{2} k x^2 = \frac{1}{2} k x^2 (1+\varepsilon - 1) = \frac{1}{2} k \varepsilon x^2 = \varepsilon V(x)$$

$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle = \varepsilon \underbrace{\langle n | V | n \rangle}_{}$$

expectation value of unperturbed potential energy in n^{th} unperturbed state

virial theorem: $\langle T \rangle = \langle V \rangle$ for stationary states harmonic oscillator

$E_n = \langle T \rangle + \langle V \rangle$

$\downarrow$

$\langle V \rangle = \frac{1}{2} E_n^0 = \frac{1}{2}\left(n + \frac{1}{2}\right) \hbar \omega$

So, $\varepsilon \langle n | V | n \rangle = \frac{\varepsilon}{2}\left(n+\frac{1}{2}\right) \hbar \omega$, which is the first-order term (derivative) from question a