Exercise 7.1

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a) $E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle = \left\langle \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \middle| \alpha \delta\left(x - \frac{a}{2}\right) \middle| \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \right\rangle$

$$= \frac{2\alpha}{a} \int_{-\infty}^{\infty} \delta\left(x - \frac{a}{2}\right) \sin^2\left(\frac{n\pi}{a}x\right) dx$$

$$= \frac{2\alpha}{a} \sin^2\left(\frac{n\pi}{a}\frac{a}{2}\right)$$

$$= \frac{2\alpha}{a} \sin^2\left(\frac{n\pi}{2}\right)$$

for even n, $\frac{n\pi}{2}$ is an integer multiple of π, which means $\sin^2\left(\frac{n\pi}{2}\right) = 0$, or $\langle H' \rangle = 0$, giving no perturbation at all

b) $\psi_1^1 = \sum_{m \neq 1} \frac{\langle \psi_m^0 | H' | \psi_1^0 \rangle}{(E_1^0 - E_m^0)} \psi_m^0$

$$= \frac{2\alpha}{a}\sqrt{\frac{2}{a}}\left(\frac{\int_{-\infty}^{\infty} \sin\left(\frac{2\pi}{a}x\right)\sin\left(\frac{\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx}{E_1^0 - E_2^0} \sin \frac{2\pi}{a}x + \frac{\int_{-\infty}^{\infty} \sin\left(\frac{3\pi}{a}x\right)\sin\left(\frac{\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx}{E_1^0 - E_3^0}\sin\left(\frac{3\pi}{a}x\right) + \frac{\int_{-\infty}^{\infty} \sin\left(\frac{4\pi}{a}x\right)\sin\left(\frac{\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx}{E_1^0 - E_4^0}\sin\left(\frac{4\pi}{a}x\right) + \frac{\int_{-\infty}^{\infty} \sin\left(\frac{5\pi}{a}x\right)\sin\left(\frac{\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx}{E_1^0 - E_5^0}\sin\left(\frac{5\pi}{a}x\right) + \frac{\int_{-\infty}^{\infty} \sin\left(\frac{6\pi}{a}x\right)\sin\left(\frac{\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx}{E_1^0 - E_6^0}\sin\left(\frac{6\pi}{a}x\right) + \frac{\int_{-\infty}^{\infty} \sin\left(\frac{7\pi}{a}\right)\sin\left(\frac{\pi}{a}x\right)\delta\left(x-\frac{a}{2}\right)dx}{E_1^0 - E_7^0}\sin\left(\frac{7\pi}{a}x\right)\right)$$

$$= \frac{2\alpha}{a}\sqrt{\frac{2}{a}}\frac{2ma^2}{\pi^2\hbar^2}\left(\frac{-1}{1-9}\sin\left(\frac{3\pi}{a}x\right) + \frac{1}{1-25}\sin\left(\frac{5\pi}{a}x\right) + \frac{-1}{1-49}\sin\left(\frac{7\pi}{a}x\right)\right)$$

$$= \frac{4\alpha m a}{\pi^2\hbar^2}\sqrt{\frac{2}{\alpha}}\left(\frac{1}{8}\sin\left(\frac{3\pi}{a}x\right) - \frac{1}{24}\sin\left(\frac{5\pi}{a}x\right) + \frac{1}{48}\sin\left(\frac{7\pi}{a}x\right)\right)$$