Exercise 5.18

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a) we have parahelium (the singlet state, in which $s=0$)
orthohelium (the triplet states, which all have $s=1$)

Thus, orthohelium has the lowest energy (lower than parahelium)

$3p$ electrons: total spin $\frac{3}{2}$ or $\frac{1}{2}$
total orbital angular momentum 3, 2, 1 or 0

b) $C: (1s)^2 (2s)^2 (2p)^2$

Why not $L=2$

- $S=1$ (1st Hund rule: highest spin)
  - $\Rightarrow$ triplet in spin $\Rightarrow$ symmetric in spin
  - $\Rightarrow$ orbital part antisymmetric

- if we were to have $L=2$, then we would have $|L\ M\rangle = |2\ M\rangle$

all states have same symmetry for all M

$|2\ 2\rangle, |2\ 1\rangle, |2\ 0\rangle, |2\ -1\rangle, |2\ -2\rangle$

all have the same symmetry

look at top of ladder

$|2\ 2\rangle = |1\ 1\rangle_1 \otimes |1\ 1\rangle_2$

$|L\ M\rangle = |l_1\ m_1\rangle_1 \otimes |l_2\ m_2\rangle_2$

$M = m_1 + m_2 = 1 + 1 = 2$ (at top of ladder)

for $L=1$, this does not happen; an antisymmetric combination of particle states is possible

$C: {}^3P_0 \Rightarrow L=1$

5.18d) for nitrogen; Hund rule #1 $\Rightarrow$ max spin $= \frac{3}{2} \Rightarrow S = \frac{3}{2}$

top of ladder: $|\frac{3}{2}\ \frac{3}{2}\rangle = |\frac{1}{2}\ \frac{1}{2}\rangle_1 |\frac{1}{2}\ \frac{1}{2}\rangle_2 |\frac{1}{2}\ \frac{1}{2}\rangle_3$ $\rightarrow$ symmetric, as exchanging two particles leads to same spin state

(with S and m_s marked on $|\frac{3}{2}\ \frac{3}{2}\rangle$)

Hund rule #2 $\Rightarrow$ max value of $L=3$, but, as spin state is already symmetric, L-state cannot be symmetric as well

$L \neq 3$

$|3\ 3\rangle = |1\ 1\rangle_1\ |1\ 1\rangle_2\ |1\ 1\rangle_3$

$|2\ 2\rangle \rightarrow$ also symmetric

$|1\ 1\rangle \rightarrow$ also symmetric

only $L=0$ will give an antisymmetric state

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alternative argument:

say electrons are in $|\frac{3}{2}\ \frac{3}{2}\rangle$ state

$\rightarrow$ orbital states of these electrons should be different

$l=1 \Rightarrow |1\ 1\rangle\ |1\ 0\rangle\ |1\ -1\rangle$ (labelled m_{l_1}, m_{l_2}, m_{l_3})

$M_L = m_{l_1} + m_{l_2} + m_{l_3} = 0$

Thus, we have to restrict $M_L = 0$ $\rightarrow$ can only be achieved for $L=0$

Hund rule #3 $J = |L - S| = |0 - \frac{3}{2}| = \frac{3}{2}$

So, the ground state N is ${}^4S_{\frac{3}{2}}$