

Extra: 2.21 (integral)

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$$\int_{-\infty}^{\infty} e^{-(ax^2 + bx)} dx =$$

$$\int_{-\infty}^{\infty} e^{-y^2} e^{-\frac{b^2}{4a}} \frac{1}{\sqrt{a}} dy$$

$$ax^2 + bx + \frac{b^2}{4a}$$

$$y^2 = a \left(x + \frac{b}{2a} \right)^2 = a \left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} \right) = ax^2 + bx + \frac{b^2}{4a}$$

$$\begin{aligned} y &= \sqrt{a} x \dots \\ dy &= \sqrt{a} dx \\ dx &= \frac{1}{\sqrt{a}} dy \end{aligned}$$