

example

$$|S m\rangle = \sum_{m_1, m_2 = m} C_{m_1, m_2, m}^{S_1, S_2, S} |S_1, S_2, m_1, m_2\rangle$$

12 1) know that 2 particles with  
 $S = \frac{3}{2}, S = \frac{1}{2}$

$$|2 1\rangle = \frac{1}{\sqrt{4}} \left| \frac{3}{2} \frac{3}{2} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle + \sqrt{\frac{3}{4}} \left| \frac{3}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

other way around  $\left| 1 \frac{1}{2} 1 -\frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}} \left| \frac{3}{2} \frac{1}{2} \right\rangle + \frac{2}{\sqrt{3}} \left| \frac{1}{2} -\frac{1}{2} \right\rangle$