Exercise 3.26

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$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle$

$|\beta\rangle = i|1\rangle + 2|3\rangle$

a) $\langle\alpha| = -i\langle 1| - 2\langle 2| + i\langle 3|$

$\langle\beta| = -i\langle 1| + 2\langle 3|$

b) $\langle\alpha|\beta\rangle = -i^2\langle 1|1\rangle + 2i\langle 3|3\rangle$

$= 1 + 2i$

$\langle\beta|\alpha\rangle = -i^2\langle 1|1\rangle - 2i\langle 3|3\rangle$

$= 1 - 2i$

So, indeed $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$

c) $\hat{A} = |\alpha\rangle\langle\beta|$

$A_{mn} = \langle m|\alpha\rangle\langle\beta|n\rangle$

$A_{11} = i\langle 1|1\rangle \cdot -i\langle 1|1\rangle = -i^2 = 1$ $\quad A_{12} = i\langle 1|1\rangle\, 0 = 0$ $\quad A_{13} = i\langle 1|1\rangle \cdot 2\langle 3|3\rangle = 2i$

$A_{21} = -2\langle 2|2\rangle \cdot -i\langle 1|1\rangle = 2i$ $\quad A_{22} = -2\langle 2|2\rangle\, 0 = 0$ $\quad A_{23} = -2\langle 2|2\rangle \cdot 2\langle 3|3\rangle = -4$

$A_{31} = -i\langle 3|3\rangle \cdot -i\langle 1|1\rangle = -1$ $\quad A_{23} = -i\langle 3|3\rangle\, 0 = 0$ $\quad A_{33} = -i\langle 3|3\rangle \cdot 2\langle 3|3\rangle = -2i$

$$\hat{A} = \begin{pmatrix} 1 & 0 & 2i \\ 2i & 0 & -4 \\ -1 & 0 & -2i \end{pmatrix}$$

A matrix with complex numbers on its diagonal is never hermitian