Exercise 4.35

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a) $\chi_+^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad \lambda = +\frac{\hbar}{2}$

$\chi_-^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad \lambda = -\frac{\hbar}{2}$

$$\chi = \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) e^{i\gamma B_0 t/2} \\ \sin\left(\frac{\alpha}{2}\right) e^{-i\gamma B_0 t/2} \end{pmatrix}$$

$$C_+^{(x)} = \chi_+^{(x)\dagger}\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) e^{i\gamma B_0 t/2} \\ \sin\left(\frac{\alpha}{2}\right) e^{-i\gamma B_0 t/2} \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\alpha}{2}\right)e^{i\gamma B_0 t/2} + \sin\left(\frac{\alpha}{2}\right)e^{-i\gamma B_0 t/2}\right)$$

invert minus signs $C_+^* C_+$

$$P_+^{(x)}(t) = |C_+^{(x)}|^2 = \frac{1}{2}\left(\cos\left(\frac{\alpha}{2}\right)e^{-i\gamma B_0 t/2} + \sin\left(\frac{\alpha}{2}\right)e^{i\gamma B_0 t/2}\right)\left(\cos\left(\frac{\alpha}{2}\right)e^{i\gamma B_0 t/2} + \sin\left(\frac{\alpha}{2}\right)e^{-i\gamma B_0 t/2}\right)$$

$$= \frac{1}{2}\left(\cos^2\left(\frac{\alpha}{2}\right) + \sin^2\left(\frac{\alpha}{2}\right) + \sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\left(e^{i\gamma B_0 t} + e^{-i\gamma B_0 t}\right)\right)$$

↓ Euler's formula disappears take outside sine

$$= \frac{1}{2}\left(1 + \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\left(\cos(\gamma B_0 t) + i\sin(\gamma B_0 t) + \cos(-\gamma B_0 t) + \sin(-\gamma B_0 t)\right)\right)$$

$$= \frac{1}{2}\left(1 + \underbrace{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\cos\gamma B_0 t\right)$$

$$= \frac{1}{2}\left(1 + \sin\alpha\cos\gamma B_0 t\right)$$

b) $\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

$$C_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \end{pmatrix}\begin{pmatrix} \cos\frac{\alpha}{2} & e^{i\gamma B_0 t/2} \\ \sin\frac{\alpha}{2} & e^{-i\gamma B_0 t/2} \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\cos\frac{\alpha}{2}e^{i\gamma B_0 t/2} - i\sin\frac{\alpha}{2}e^{-i\gamma B_0 t/2}\right)$$

$$P_+^{(y)} = |C_+^{(y)}|^2 = \frac{1}{2}\left(\cos\frac{\alpha}{2}e^{-i\gamma B_0 t/2} + i\sin\frac{\alpha}{2}e^{i\gamma B_0 t/2}\right)\left(\cos\frac{\alpha}{2}e^{i\gamma B_0 t/2} - i\sin\frac{\alpha}{2}e^{-i\gamma B_0 t/2}\right)$$

$$= \frac{1}{2}\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\left(e^{i\gamma B_0 t} - e^{-i\gamma B_0 t}\right)\right)$$

$$\downarrow \cos\gamma B_0 t + i\sin\gamma B_0 t - \cos(-\gamma B_0 t) - i\sin(-\gamma B_0 t)$$

$$\downarrow 2i\sin\gamma B_0 t$$

$$= \frac{1}{2}\left(1 + i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\ 2i\sin\gamma B_0 t\right)$$

$$= \frac{1}{2}\left(1 - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\sin\gamma B_0 t\right)$$

$$= \frac{1}{2}\left(1 - \sin\alpha\sin\gamma B_0 t\right)$$

c) $\chi_+^{(z)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$C_+^{(z)} = \begin{pmatrix} 1 & 0 \end{pmatrix}\chi = \cos\frac{\alpha}{2}e^{i\gamma B_0 t/2}$$

$$P_+^{(z)}(t) = |C_+^{(z)}|^2 = \cos\frac{\alpha}{2}e^{-i\gamma B_0 t/2}\cos\frac{\alpha}{2}e^{i\gamma B_0 t/2}$$

$$= \cos^2\frac{\alpha}{2}$$