Exercise 4.23

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equation 3.73 Hamiltonian $= 0$

$$\frac{d\langle L_x\rangle}{dt} = \frac{i}{\hbar}\langle [H, L_x]\rangle + \left\langle \frac{\partial L_x}{\partial t}\right\rangle$$

$$[H, L_x] = \left[\frac{p^2}{2m} + V, L_x\right]$$

$$= \left[\frac{p^2}{2m}, L_x\right] + [V, L_x]$$

4.22c

$$= 0 + [V, L_x]$$

$$= [V, y p_z - z p_y]$$

$$= [V, y p_z] - [V, z p_y]$$

$= 0$, as ... ?

$$= y[V, p_z] + [V, y]p_z - z[V, p_y] - [V, z]p_y$$

3.14c

$$= y\, i\hbar \frac{\partial V}{\partial z} - z\, i\hbar \frac{\partial V}{\partial y}$$

$$= i\hbar\left(y\frac{\partial V}{\partial z} - z\frac{\partial V}{\partial y}\right)$$

$$= i\hbar\,\big(\mathbf{r} \times (\nabla V)\big)_{x\text{-component}}$$

Permutate for L_y and L_z to obtain the same for the other components

Then, we get

$$\frac{d\langle \mathbf{L}\rangle}{dt} = \frac{i}{\hbar}\big\langle i\hbar\,(\mathbf{r}\times(\nabla V))\big\rangle$$

$$= i^2 \langle \mathbf{r} \times (\nabla V)\rangle$$

$$= \langle \mathbf{r} \times (-\nabla V)\rangle$$

$$= \langle \mathbf{N}\rangle$$

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b) If $V(\mathbf{r}) = V(r)$, then $\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}}$, but, that implies

(also) just a component in $\hat{r}$ $\Longrightarrow \mathbf{r}\times\hat{r} = 0$

$$\frac{d\langle \mathbf{L}\rangle}{dt} = \left\langle \mathbf{r} \times \left(-\frac{\partial V}{\partial r}\hat{r}\right)\right\rangle$$

$$= \langle 0 \rangle$$

$$= 0$$