Exercise 4.22

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$[L_z, x] = (L_z x - x L_z) f = ((x p_y - y p_x) x - x(x p_y - y p_x)) f$

$=$

eq. (3.64)

$[L_z, x] = [x p_y - y p_x, x] = [x p_y, x] - [y p_x, x]$

$= x[p_y, x] + [x, x] p_y - y[p_x, x] - [y, x] p_x$

$= 0 + 0 - y(-i\hbar) - 0$

$= -i\hbar y$

$[L_z, y] = [x p_y - y p_x, y] = [x p_y, y] - [y p_x, y]$

$= x[p_y, y] + [x, y] p_y - y[p_x, y] - [y, y] p_x$

$\quad 0 \quad 0 \quad 0$

$= -i\hbar x$

$[L_z, z] = [x p_y - y p_x, z] = [x p_y, z] - [y p_x, z] = 0$

$[L_z, p_x] = [x p_y - y p_x, p_x] = [x p_y, p_x] - [y p_x, p_x]$

no combination of x and p_x

$= x[p_y, p_x] + [x, p_x] p_y - 0$

$= i\hbar p_y$

$[L_z, p_y] = [x p_y - y p_x, p_y] = [x p_y, p_y] - [y p_x, p_y]$

$= 0 - y[p_x, p_y] - [y, p_y] p_x$

$= -i\hbar p_x$

$[L_z, p_z] = [x p_y, p_z] - [y p_x, p_z] = 0 - 0 = 0$

b) $[L_z, L_x] = [x p_y - y p_x, y p_z - z p_y]$

$= [x p_y, y p_z - z p_y] - [y p_x, y p_z - z p_y]$

$= [L_z, y p_z - z p_y]$

$= [L_z, y p_z] - [L_z, z p_y]$

$= [L_z, y] p_z + y[L_z, p_z] - [L_z, z] p_y - z[L_z, p_y]$

$= -i\hbar x p_z + 0 - 0 + z i\hbar p_x$

$= i\hbar (z p_x - x p_z)$

$= i\hbar L_y$

c) $[L_z, r^2] = [L_z, x^2] + [L_z, y^2] + [L_z, z^2]$

$= x[L_z, x] + [L_z, x] x + y[L_z, y] + [L_z, y] y + z[L_z, z] + [L_z, z] z$

$= x i\hbar y + i\hbar y x - y i\hbar x - i\hbar x y + 0 + 0$

$= 0$

$[L_z, p^2] = [L_z, p_x] p_x + p_x [L_z, p_x] + [L_z, p_y] p_y + p_y [L_z, p_y] + p_z [L_z, p_z] + [L_z, p_z] p_z$

p_i commutes for $i \neq j$

$= i\hbar p_y p_x + p_x i\hbar p_y - i\hbar p_x p_y - p_y i\hbar p_x + 0 + 0$

$= 0$

d) (By permutating x, y, z in question c) it follows that all components of $\underline{L}$ commute with p^2 and r^2.

as $H = \frac{p^2}{2m} + V(r)$

$= \frac{p^2}{2m} + V(\sqrt{r^2})$, H only depends on p^2 and r^2, and thus H commutes with $\underline{L}$.