Exercise 3.38

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$$\Delta t = \frac{\tau}{\pi}$$

$$\Psi(x,0) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + \psi_2(x)\right)$$

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1(x) e^{-iE_1 t/\hbar} + \psi_2(x) e^{-iE_2 t/\hbar}\right)$$

orthogonal

$$\langle \Psi(x,t) | \Psi(x,0) \rangle = 0$$

conjugate

$$\frac{1}{2}\left(e^{+iE_1 t/\hbar}\langle\psi_1|\psi_1\rangle + \underbrace{e^{iE_1 t/\hbar}\langle\psi_1|\psi_2\rangle + e^{iE_2 t/\hbar}\langle\psi_2|\psi_1\rangle}_{=0 \text{ (orthogonal)}} + e^{iE_2 t/\hbar}\langle\psi_2|\psi_2\rangle\right) = 0$$

$$\frac{1}{2}\left(e^{iE_1 t/\hbar} + e^{iE_2 t/\hbar}\right) = 0$$

$$-e^{iE_1 t/\hbar} = e^{iE_2 t/\hbar}$$

$$-1 = e^{i(E_2 - E_1)t/\hbar}$$

$$e^{i(2k+1)\pi} = e^{i(E_2 - E_1)t/\hbar}$$

first time for $k=0$

$$e^{i\pi} = e^{i(E_2 - E_1)\tau/\hbar}$$

$$\pi = \frac{(E_2 - E_1)\tau}{\hbar}$$

$$\tau = \frac{\pi\hbar}{E_2 - E_1}$$

$$\Delta t = \frac{\tau}{\pi} = \frac{\hbar}{E_2 - E_1}$$

It seems $\Delta E = \sigma_H = \frac{1}{2}(E_2 - E_1)$

Then $\Delta t \Delta E = \frac{\hbar}{2}$