Exercise 3.25

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This is problem 3.23 in the second edition.

For an eigenvalue E of $\hat{H}$, we have

$\hat{H}|\psi\rangle = E|\psi\rangle$, where $|\psi\rangle$ is the eigenvector corresponding to E

Let $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$

Then
$$\hat{H}|\psi\rangle = \epsilon\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)\left(c_1|1\rangle + c_2|2\rangle\right)$$
$$= \epsilon\left(c_1|1\rangle - c_2|2\rangle + c_2|1\rangle + c_1|2\rangle\right) \quad \langle 2|2\rangle = 1$$
$$= \epsilon\left((c_1 + c_2)|1\rangle + (c_1 - c_2)|2\rangle\right)$$
$$= E|\psi\rangle = E\left(c_1|1\rangle + c_2|2\rangle\right)$$

$$\epsilon(c_1 + c_2) = Ec_1 \qquad \epsilon(c_1 - c_2) = Ec_2$$

$$c_2 = \frac{Ec_1 - \epsilon c_1}{\epsilon} = \left(\frac{E}{\epsilon} - 1\right)c_1 \qquad c_1 = \frac{Ec_2 + \epsilon c_2}{\epsilon} = \left(\frac{E}{\epsilon} + 1\right)c_2$$

$$c_2 = \left(\frac{E}{\epsilon} - 1\right)\left(\frac{E}{\epsilon} + 1\right)c_2$$
$$1 = \left(\frac{E}{\epsilon}\right)^2 - 1$$
$$\left(\frac{E}{\epsilon}\right)^2 = 2$$
$$\frac{E}{\epsilon} = \pm\sqrt{2}$$
$$E = \pm\sqrt{2}\,\epsilon$$

$$c_2 = \frac{Ec_1 - \epsilon c_1}{\epsilon} = \frac{\pm\sqrt{2}\,\epsilon c_1 - \epsilon c_1}{\epsilon} = \left(\pm\sqrt{2} - 1\right)c_1$$

c_1 is an arbitrary number

$$|\psi_\pm\rangle = c_1\left(|1\rangle + (\pm\sqrt{2} - 1)|2\rangle\right)$$

explanation in lecture 4

$$H_{mn} = \langle m|\hat{H}|n\rangle \quad ; \quad H = \epsilon\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$$

$$H_{11} = \langle 1|\hat{H}|1\rangle = \epsilon\left(\langle 1|1\rangle\langle 1|1\rangle - \langle 1|2\rangle\langle 2|1\rangle + \langle 1|1\rangle\langle 2|1\rangle + \langle 1|2\rangle\langle 1|1\rangle\right)$$
$$= \epsilon$$

$$H = \epsilon\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$