Exercise 2.41

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$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2, & x>0\\ \infty, & x<0\end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V\psi=E\psi$$

when $x<0$, $V=\infty \Rightarrow \psi(x)=0$ for $x<0$

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi$$

for $x>0$

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+\frac{1}{2}m\omega^2x^2\Psi$$

for $x<0$ $\quad i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+\infty\Psi$

this implies that for $x<0$, $\Psi(x,t)=0$ otherwise, this equation would go to ∞ on the right side

this gives us a boundary condition that $\Psi(0,t)=0$, as the wavefunction must be continuous

for the other part, we essentially get the same reasoning as for the whole harmonic oscillator, except that even solutions $(n=0,2,4..)$ are invalid, as they do not satisfy the boundary condition. Thus, we get $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$ with $n=1,3,5\ldots$