$$a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$$
$$a_-\psi_n = \sqrt{n}\,\psi_{n-1}$$

$$a_+ = \frac{1}{\sqrt{2\hbar m\omega}}\left(-i\hat{p} + m\omega\hat{x}\right)$$
$$a_- = \frac{1}{\sqrt{2\hbar m\omega}}\left(+i\hat{p} + m\omega\hat{x}\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}\,(a_+ + a_-)$$
$$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}\,(a_+ - a_-)$$

$$\Delta l = l' - l = \pm 1$$
$\Delta m = m' - m = 0$ in z-direction
$\Delta m = m' - m = \pm 1$ in xy-direction

$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle$$
$$\psi_n^1 = \sum_{m\neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}\,\psi_m^0$$
$$E_n^2 = \sum_{m\neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}$$

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$
$$E\Psi = H\Psi$$

separated solution part
$$\psi(t) = e^{-iEt/\hbar}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$E_{Bohr} = \frac{E_1}{n^2};\quad E_1 = -13.6\,\text{eV}$$

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$$
exp. value of commutator of A and B

$$\sigma_x\sigma_p \geq \frac{\hbar}{2}$$
$$[\hat{x},\hat{p}] = i\hbar$$
$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$
$$[\hat{A}+\hat{B},\hat{C}] = [\hat{A},\hat{C}] + [\hat{B},\hat{C}]$$
$$[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B}$$

quantum dynamics

$$|\Psi\rangle = \sum_{n=1}^{\infty} c_n e^{-iE_nt/\hbar}|\psi_n\rangle \quad \text{and } H|\psi_n\rangle = E|\psi_n\rangle$$

1. take $\Psi(t) = c_a(t)\psi_a e^{-iE_at/\hbar} + c_b(t)\psi_b e^{-iE_bt/\hbar}$
2. solve Schr. eq. $\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ with $\hat{H} = \hat{H}^0 + \hat{H}'(t)$
3. find $\frac{dc_a}{dt}$ as function of c_b and $\frac{dc_b}{dt}$ as function of c_a
4. in zeroth order, take $c_a^{(0)}(t) = c_a(0)$ and $c_b^{(0)}(t) = c_b(0)$
5. calculate higher order by substituting one order lower into DE, then integrating

in the book, first order

transition probability: $P_{a\to b}(t) = |c_b(t)|^2$ if $c_a(0) = 1$

stimulated emission → one photon absorbed, two emitted

spontaneous emission rate: $A = \frac{\omega_0^3|\wp|^2}{3\pi\epsilon_0\hbar c^3}$

lifetime of state $\tau = 1/A$

$$\int_0^\infty x^n e^{-x/a}\,dx = n!\,a^{n+1}$$
$$\int_0^\infty x^{2n} e^{-x^2/a^2}\,dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$$
$$\int_0^\infty x^{2n+1} e^{-x^2/a^2}\,dx = \frac{n!}{2}\,a^{2n+2}$$

gaps → forbidden E
bands → allowed E

partially filled band → conductor (next E is 'close')

completely filled band → insulator

$$L_x = yp_z - zp_y$$
$$[L_x, L_y] = i\hbar L_z$$
$$L_\pm = L_x \pm iL_y$$
$$[L^2, L_x] = 0$$
$$[L_z, L_\pm] = \pm\hbar L_\pm$$
$$[L^2, L_\pm] = 0$$
$$S^2|sm\rangle = \hbar^2 s(s+1)|sm\rangle$$
$$S_z|sm\rangle = \hbar m|sm\rangle$$

Fermi energy:

$$E_F = \frac{\hbar^2}{2m}(3\rho\pi^2)^{2/3}$$

Fermi surface separates occupied and unoccupied states

Fermi energy is difference in kinetic energy between highest excited state and ground state

bosons are closer together (integer spin)
fermions are further apart
↳ (have half-integer spin)

s = 1: triplet: symmetric
s = 0: singlet: antisymmetric

parahelium: antisymmetric spin
orthohelium: symmetric spin

element	configuration	
H	(1s)	$^2S_{1/2}$
He	$(1s)^2$	$^1S_0$
Li	(He)(2s)	$^2S_{1/2}$
Be	$(He)(2s)^2$	$^1S_0$
B	$(He)(2s)^2(2p)$	$^2P_{1/2}$
C	$(He)(2s)^2(2p)^2$	$^3P_0$
N	$(He)(2s)^2(2p)^3$	$^4S_{3/2}$
O		
Ne		

l =	name
0	s
1	p
2	d
3	f

Na	(Ne)(3s)	$^2S_{1/2}$
Mg	$(Ne)(3s)^2$	$^1S_0$
Al	$(Ne)(3s)^2(3p)$	$^2P_{1/2}$
Si	$(Ne)(3s)^2(3p)^2$	$^3P_0$
P	$(Ne)(3s)^2(3p)^3$	$^4S_{3/2}$

good quantum numbers

fine struc/rel. correction → n, l, m

fine struc/spin-orbit coupling → l, s, j, m_j

and use $J^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S}$, which gives $\vec{L}\cdot\vec{S} = \frac{1}{2}(J^2 - L^2 - S^2)$

fine struc/total → n, l, s, j, m_j

weak-field ZME: n, l, j, m_j

strong-field ZME: n, l, m_l, m_s

$$W_{ij} = \langle \psi_i^0 | H' | \psi_j^0 \rangle$$

eigenvalues of matrix W give first-order energy corrections

eigenvectors give good states

$$S = \frac{\hbar}{2}\sigma$$
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

WKB

amplitude phase

$\psi(x) = A(x)e^{i\phi(x)}$

$p(x) = \sqrt{2m(E-V(x))}$

integral over well

$\psi(x) = \frac{C}{\sqrt{p(x)}} e^{\pm\frac{i}{\hbar}\int p(x)dx}$ $(E>V)$

$\psi(x) = \frac{C}{\sqrt{|p(x)|}} e^{\pm\frac{1}{\hbar}\int |p(x)|dx}$ $(E<V)$

integral over barrier

$T \sim e^{-2\gamma}$ with $\gamma = \frac{1}{\hbar}\int_0^a |p(x)|\,dx$

- use tunnel factor instead of phase for tunneling
- phase = integral over local wave number
- works good far from classical turning points
- determine bound state energies using:

\# vertical walls

$= 0 \rightarrow \int_{well} p(x)dx = (n-\frac{1}{2})\pi\hbar$

$= 1 \rightarrow \int_{well} p(x)dx = (n-\frac{1}{4})\pi\hbar$

$= 2 \rightarrow \int_{well} p(x)dx = n\pi\hbar$

∞ square well

$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$ $\qquad \psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$

harmonic oscillator

$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2}$

$\psi_n(x) = \left((\hat{a}_+)^n \psi_0(x)\right)\cdot\frac{1}{\sqrt{n!}}$

$E_n = (n+\frac{1}{2})\hbar\omega$

δ-function potential

(for a potential well:) $V(x) = -\alpha\delta(x)$

$E = -\frac{m\alpha^2}{2\hbar^2}$

$\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2}$

j goes from $l+s$ to $|l-s|$ in integer steps

for some operator $\hat{Q}$:

$\hat{Q}$'s eigenfunctions are a complete set $\rightarrow \hat{Q}|f_n\rangle = q_n|f_n\rangle \rightarrow |\psi\rangle$ can be changed to the basis of $\hat{Q}$-eigenfunctions $\rightarrow |\psi\rangle = \sum_n d_n|f_n\rangle$

then, probability of measuring q_n is given by $|d_n|^2$.

The coefficient $\langle x|n\rangle = f_n(x)$ is a basis wavefunction; expansion of wavefunction $\psi(x) = \langle x|\psi\rangle = \sum_n \langle x|n\rangle\langle n|\psi\rangle = \sum_n c_n f_n(x)$

Variational principle

finds upper bounds for GS energy

1. pick a trial wavefunction ψ with variational parameters $b_1, \ldots, b_n$
2. normalize ψ (if not done yet)
3. determine the expectation value of the hamiltonian as function of b_i's
4. find the values of b_i that minimize E_{GS} using $\frac{\partial E}{\partial b_i} = 0$
5. fill in these values for b_i to obtain an upper bound for E_{GS}

example variational principle: $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2$

$\psi(x) = Ae^{-bx^2} \longrightarrow 1 = |A|^2\int_{-\infty}^{\infty} e^{-2bx^2}dx = |A|^2\sqrt{\frac{\pi}{2b}} \Rightarrow A = \left(\frac{2b}{\pi}\right)^{\frac{1}{4}}$

$\langle H\rangle = \langle T\rangle + \langle V\rangle$

$\langle T\rangle = -\frac{\hbar^2}{2m}|A|^2\int_{-\infty}^{\infty} e^{-bx^2}\frac{d^2}{dx^2}\left(e^{-bx^2}\right)dx = \frac{\hbar^2 b}{2m}$

⋮